Să se demonstreze deducţiile următoare utilizând o strategie/rafinare a rezoluţiei:

(∀x)(¬P(x) ∧ ¬Q(x)→R(x)), (∀y)(R(y) → W(y)), (∀x)(W(x) → P(x)), ¬P(a), ¬W(c) ⊢ (∃z)Q(z)

U1= (∀x)(¬P(x) ∧ ¬Q(x)→R(x))

Aducem la forma normala prenexa:

Pas 1: Se inlocuiesc conectivele → si ↔ folosind ¬,∧,∨

U1= (∀x)(¬(¬ P(x) ∧ ¬Q(x)) ∨ R(x))

Pasul 2: Se aplica legile finite si infinite ale lui DeMorgan astfel incat cuantificatorii sa nu fie precedati de negatie

U1= (∀x)((P(x) ∨ Q(x)) ∨ R(x))

U1= (∀x)(P(x) ∨ Q(x) ∨ R(x))

Pasul 3: Se redenumesc variabilele legate astfel incat ele sa fie distincte

U1= (∀x)(P(x) ∨ Q(x) ∨ R(x))

Pasul 4: Se utilizeaza echivalentele logice care reprezinta legile de axtragere a cuantificatorilor in fata formulei

U1= (∀x)(P(x) ∨ Q(x) ∨ R(x))

Aducem la forma normala Skolem:

U1= (∀x)(P(x) ∨ Q(x) ∨ R(x))

Eliminam cuantificatorii existantiali:

Nu avem

U1s= (∀x)(P(x) ∨ Q(x) ∨ R(x))

Aducem la forma normala Skolem fara cuantificatori:

U1s= (∀x)(P(x) ∨ Q(x) ∨ R(x))

Eliminam cuantificatorii universali:

U1Sq= P(x) ∨ Q(x) ∨ R(x)

Aducem la forma normala clauzala:

U1Sq= P(x) ∨ Q(x) ∨ R(x)

U1C = P(x) ∨ Q(x) ∨ R(x)=C1

U2= (∀y)(R(y) → W(y))

U2= (∀y)(¬ R(y) ∨ W(y))

U2s= (∀y)(¬ R(y) ∨ W(y))

U2Sq= ¬ R(y) ∨ W(y)

U2C = ¬ R(y) ∨ W(y) =C2

U3= (∀x)(W(x) → P(x))

U3= (∀x)(¬ W(x) ∨ P(x))

U3s= (∀x)(¬ W(x) ∨ P(x))

U3Sq= ¬ W(x) ∨ P(x)

U3C = ¬ W(x) ∨ P(x) =C3

U4=¬P(a)=C4

U5=¬W(c)=C5

V=(∃z)Q(z)

¬ V= ¬ (∃z)Q(z)

¬ V= (∀z) ¬ Q(z)= (¬ V)s

(¬ V)Sq= ¬ Q(z)

(¬ V)C= ¬ Q(z)=C6

S={P(x) ∨ Q(x) ∨ R(x), ¬ R(y) ∨ W(y) , ¬ W(x) ∨ P(x), ¬P(a), ¬W(c), ¬ Q(z)}

C1 = P(x) ∨ Q(x) ∨ R(x)

C2=¬R(y) ∨ W(y)

C3= ¬W(x) ∨ P(x)

C4= ¬P(a)

C5= ¬W(c)

C6= ¬Q(z)

Rezolutia liniara:

Θ=[x ←a]

C7=ResΘPr(C1,C4)= Q(a) ∨ R(a)

λ=[z ←a]

C8= Res λPr(C7,C6)=R(a)

ϕ=[y ←a]

C9= ResϕPr(C8,C2)=W(a)

Θ1=[x ←a]

C10= ResΘ1Pr(C9,C3)= P(a)

C11= ResPr(C10,C4)= □

S este inconsistenta ⇒ (∀x)(¬P(x) ∧ ¬Q(x)→R(x)), (∀y)(R(y) → W(y)), (∀x)(W(x) → P(x)), ¬P(a), ¬W(c) ⊢ (∃z)Q(z)